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Numerical Simulation of Biomagnetic Fluid in a Channel with Thrombus

Haik, Y.*, Chen, C.-J.* and Chatterjee, J.*

* FAMU-FSU College of Engineering, Tallahassee, Florida 32310, USA.
e-mail: haik@eng.fsu.edu

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***Abstract*:** Biomagnetic fluid dynamics is the study of the interaction of biological fluids with an applied steady magnetic field. Recently, several medical applications begin to utilize magnetic labeling of specific cells and targeted drug delivery using magnets. The magnetically labeled cells and the drug encapsulates are usually loaded in the blood stream and are directed toward a specific site by use of a magnet. In this paper, numerical simulation of biomagnetic fluid in the presence of a thrombus when exposed to magnetic field is presented. The finite analytic method is used to obtain the numerical simulation. It is found that the magnetic force causes a drastic change in the fluid behavior and the friction coefficient increases as the magnetic field strength increases.

***Keywords*:** non-linear sloshing, moving boundary, body-fitted coordinates, arbitrary Lagrangian-Eulerian, numerical visualization.

1. Introduction

In the biomagnetic fluid dynamics, the biofluid has magnetic characteristics and is able to interact with an applied magnetic field. The applied magnetic field is steady, thus the applied magnetic field is interacting with the biomagnetic fluid through the magnetization effect. In general, biological molecules and cells are considered diamagnetic materials. Diamagnetic materials pull away from the strong magnetic field. Levitation of biological materials such as blood was demonstrated in a high magnetic field. Several investigators reported the effect of high magnetic field on the cellular components of the biological fluids. Ichioka et al. (2000) recently studied the effect of high magnetic field on skin blood flow in a rat subjected to high magnetic field using laser doppler flowmeter. They found that the flow rate slows down when the rat is exposed to a field of 8 tesla. Haik et al. (1999) reported similar results for blood flow in vitro. Higashi et al. (1993) studied the orientation of the erythrocytes in strong magnetic field, from 1 to 8 tesla, and reported that the erythrocytes are found to orient with their disk plane parallel to the magnetic field. Motta et al (1998) studied the effect of magnetic field on the human whole blood light absorption. Their results showed that there was an orientational effect on the blood cells due to the applied magnetic field. The orientational effect due to the magnetic field was found to enhance the fluid viscosity (Hall and Busenberg, 1969; McTague, 1969). It has been found that human red blood cells have the characteristics of a paramagnetic fluid when deoxygenated and diamagnetic when oxygenated (Haik et al. 1999). The biological elements whether diamagnetic or paramagnetic in general have a weak magnetic susceptibility in the order of 10^{-6}. In order to perform mechanical work on the biological fluids, such as separation of cellular components, the magnetization of the biofluids are first enhanced by tagging the cellular or molecular components with magnetic material (Lubbe et al., 1998; Zobrowski et al., 1998 and Hafeli et al., 1998).

Labeled encapsulated magnetic particles are introduced in the biofluids stream and are used in many clinical

applications. These encapsulated magnetic particles are spherical in shape and generally are referred to as the magnetic microspheres. In using the magnetic microspheres for medical applications, the surface of the magnetic microspheres is treated so that if a target such as red cells, polymer or protein is present, the magnetic microspheres will hybridize to it.

The production of specialized magnetic microspheres for different medical applications has become a major research thrust in recent years. In our laboratory we have developed magnetic particles in the order of 2 nm in diameter. These particles at this small size are considered biodegradable and can be used in medical applications. When the particles are introduced to the biofluid stream, the fluid behaves as a magnetic fluid. Figure 1 shows a red blood cell attached to a magnetic particle. The labeling of cellular components with magnetic particles is being used in many medical applications such as cellular separation and targeted drug delivery. This paper presents a numerical modeling of biomagnetic fluid flow in a channel that has a semi-circular thrombus and an externally imposed magnetic field.

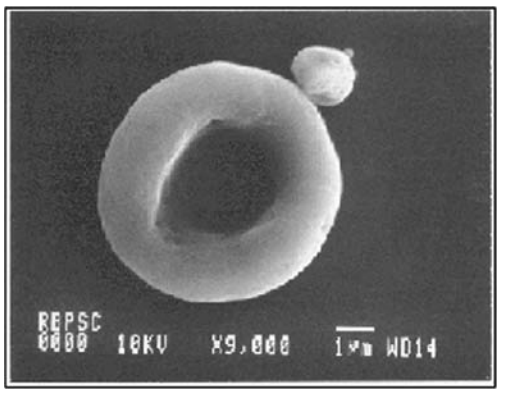

Fig. 1. Red blood cell attached to magnetic particle.

2. Mathematical Model

The mathematical model for the biomagnetic fluid dynamics is based on the modified Stokes principles. From the modified principles of constitutive relations, the equations of motion for incompressible flow are derived (Haik et al. 1999). They are

Continuity Equation:

$$\nabla \bullet \vec{V} = 0 \tag{1}$$

Linear Momentum Equation:

$$\rho \frac{D\vec{V}}{Dt} = \nabla P + \rho \vec{F} + (\mu + \kappa)\nabla^2 \vec{V} + \kappa\left(\nabla^2 \vec{V} + 2\nabla \times \vec{\omega}\right) + \mu_0 \left(\vec{M} \bullet \nabla\right)\vec{H} \tag{2}$$

Angular Momentum Equation:

$$\rho I \frac{D\vec{\omega}}{Dt} = \mu_0 \vec{M} \times \vec{H} + (\mu' + \alpha)\nabla^2 \vec{\omega} + 2\kappa\left(\nabla \times \vec{V} - 2\vec{\omega}\right) \tag{3}$$

The linear momentum equation above shows two additional terms to the Navier-Stokes equation for Newtonian fluids to include the effect of magnetic field on the fluid. The first term, $\kappa\left(\nabla^2 \vec{V} + 2\nabla \times \vec{\omega}\right)$ of the two additional terms, models the anti-symmetric part of the stress tensor. This is because when the biomagnetic fluid is subjected to a magnetic field, the action of magnetization will introduce a rotational motion to orient the magnetic fluid particles with the magnetic field. This action of orientation produces anti-symmetric stress tensor, which is directionally dependent of fluid particle rotation and the rotation of the flow field.

The second term, $\mu_0\left(\vec{M} \bullet \nabla\right)\vec{H}$, represents the magnetic force due to polarization. This stress depends on the existence of the magnetic field gradient. When the magnetic field gradient is absent, this force vanishes.

Based on theoretical study (Haik et al. 2001) the orientation time for cellular components in the biofluid is typically of order of one second or less which is much less than the average time scale of the biofluid motion in human. Further, the time scale of orientation for the cellular and biological components coupled with magnetic particles, is dependent on the orientation time for the magnetization vector of the magnetic particles. The magnetic particles orient with the applied magnetic field at a very short time compared with the average fluid flow time scale. The theoretical calculations were in agreement with the experimental observations of Higashi et al. (1993). Experiments also (Haik et al., 2001) showed that the magnetization could be modeled as

$$\vec{M} = c\vec{H}$$

where χ is the average magnetic susceptibility of the biofluid. χ is dimensionless and is determined experimentally. A simplified mathematical model can be obtained by coupling the orientation effects to the shear stress via the apparent viscosity. In order to accommodate for the orientation effect the viscosity needs to be adjusted to describe the effect of the magnetic field on the biofluid flow. In the magnetic viscosity model the linear momentum reduces to

$$r\frac{D\vec{V}}{Dt} = \nabla P + r\vec{F} + h^{*}\nabla^{2}\vec{V} + m_{o}c\nabla H^{2} \qquad (4)$$

where η^{*} is the apparent viscosity. The effective viscosity is determined by experiments. In the case where the cellular component is attached to the magnetic material, the apparent viscosity approaches the kinematic viscosity of a colloid solution.

The approximated model of the biomagnetic fluid reduces the angular momentum equation to the algebraic expression of the equality of the symmetry of the stress tensor. This modification is considered valid especially when the cellular components are subjected to high magnetic field or are attached to superparamagnetic particles.

In the biomagnetic fluid the magnetic field is affecting the fluid through the action of magnetization. In this case the magnetic field equations can be written as

$$\vec{B} = m_{o}(1 + c)\vec{H}$$

$$\nabla \bullet B = \nabla \times H = 0$$

The dimensionless form of the equation of motion with the pressure term including the hydrostatic pressure from the gravitational force can be written as

$$\frac{D\vec{V}}{Dt} = \nabla P + \frac{1}{Re}\nabla^{2}\vec{V} + \frac{Mn}{2}\nabla B^{2} \qquad (5)$$

$$Re = \frac{rV_{o}L}{h^{*}}$$

$$Mn = \frac{cB_{o}^{2}}{m_{o}rV_{o}^{2}}$$

where Re is the Reynolds number and Mn is the magnetic number. Here V_o is the reference velocity, and μ_o is the magnetic permeability.

3. Numerical Simulation

The finite analytic method (Chen et al., 2000) was used to numerically simulate the fluid flow in a two-dimensional channel flow with a semi-circular thrombus. In the finite analytic method the transport equation can be modeled as

$$R\left(f_{l} + uf_{x} + vf_{y}\right) = f_{xx} + f_{ww} + S$$

In the present study ϕ is the velocity, R is the Reynolds number and S is the source term. In the momentum equation (5) for the biomagnetic fluid flow the source term represents the pressure gradient and the magnetic force.

The computational domain is shown in Fig. 2. The thrombus radius is normalized to one and the channel height is 3 times the semi-circular thrombus radius. The channel length is chosen to be 25 times of the thrombus radius. The center of the thrombus is located at $X = 6$ and the magnet is located between $X = 4$ and $X = 6$ at the

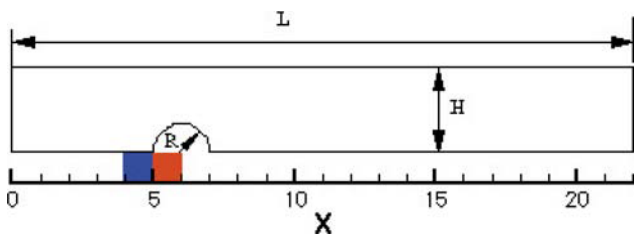

Fig. 2. Computational domain.

bottom of the thrombus.

A nonuniform grid distribution is used, the grids are enhanced at the thrombus location. For the discretization involving the thrombus, the diagonal Cartesian method was used (Chen et al., 1995; Lin, 1997). In this method the complex boundaries are approximated by both Cartesian grid lines and diagonal lines segments in the Cartesian coordinates. For more details of the method employed refer to Lin (1997) and Carlson (1997).

The maximum magnetic field formed from two strong magnets shown in Fig. 2 was placed at $X = 5.0$ which is the beginning of the thrombus. The red color is the north facing magnet while the blue is the south facing magnet. The height to radius ratio is 3. The length to radius ratio is 26. The dimensionless magnetic field distribution is shown in Fig. 3, the nondimensional magnetic force in the X and Y directions is shown in Figs. 4 and 5 respectively. The origin of the X axis shown in Figs. 3-5 corresponds to the magnets, where $X = 0$ represents $X = 5$ from the channel edge. The magnetic field gradients are shown in Fig. 3.

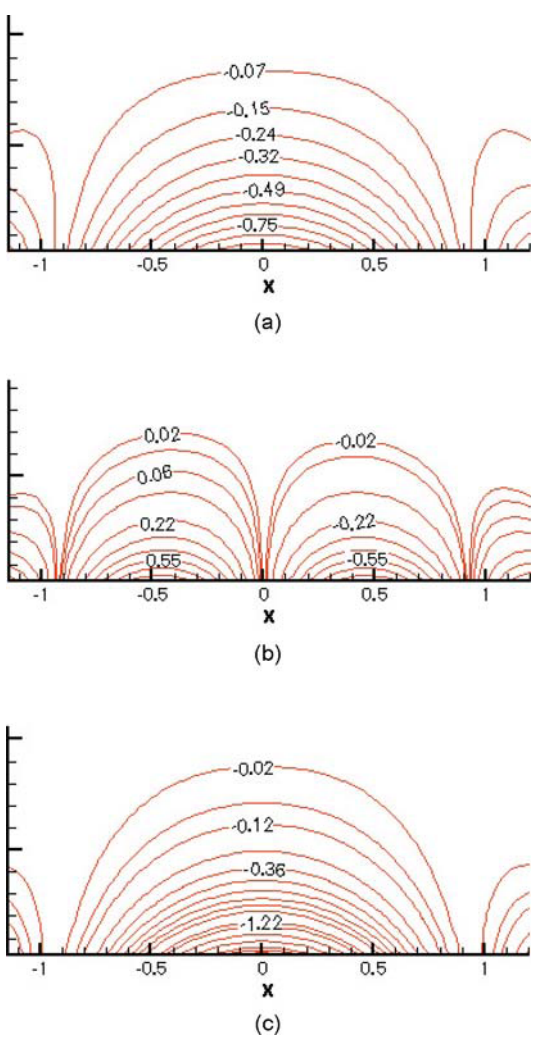

Fig. 3. (a) Nondimensional magnetic field contours (b) Magnetic force in the *x* direction (c) Magnetic force in the *y* direction.

As shown from Fig. 3 the magnetic field is strongest next to the poles. The magnetic force which is proportional to the magnetic field and the magnetic field gradient is strongest near the poles. The alternating pole arrangement causes an increase in the magnetic force at the joint location of the alternating poles.

Consider a biomagnetic fluid that consists of nano magnetic particles attached to biological cells (Chatterjee et al., 2001). The biomagnetic fluid is flowing in a vessel of 3 mm in height with a thrombus of 1 mm radius. The magnetic susceptibility of the biomagnetic fluid is $\chi = 10^{-4}$. The apparent viscosity of the biomagnetic fluid is 5.2 times higher than the water because of the cells and the magnetic particles. A parabolic inlet velocity profile is applied at the inlet and a fully developed assumption is applied at the exit. Three different magnetic field strength are studied namely (B_o = 0, 1.3, 1.7) and two Reynolds numbers (Re = 10 and Re = 100) based on the vessel height and the maximum inlet velocity. Table below shows the corresponding Reynolds number and Magnetic number for the cases that are studied.

Table 1.

Re	V_o (mm/sec)	B_o(T)	Mn
10	17.3	0	0
		1.3	449
		1.7	768
100	173	0	0
		1.3	4.49

The stream functions at different magnetic fields for Re = 10 are shown in Figs. 4, 5 and 6. Without magnetic field a small recirculation zone is formed just behind to the thrombus downstream boundary. When the magnetic field is imposed on the flow the rear recirculation zone grows substantially in size and another recirculation zone is formed at the upstream edge of the thrombus. The strength of the recirculation increases as the magnetic field increases. The physical reason behind this behavior is that the magnetic force accelerates the fluid toward the thrombus and thus slow the flow near the upper wall. Also once the fluid reaches the thrombus it is reflected upward with a relatively high velocity. Consequently this induces the upstream separation and thus recirculation.

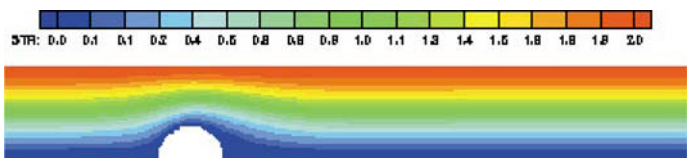

Fig. 4. Stream functions for Re = 10 and B_o = 0.

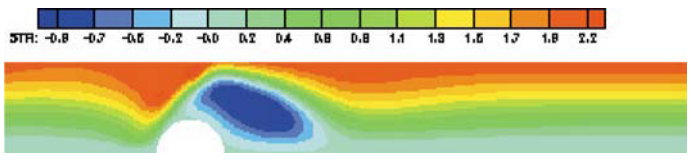

Fig. 5. Stream functions for Re = 10, B_o = 1.3 and Mn = 449.

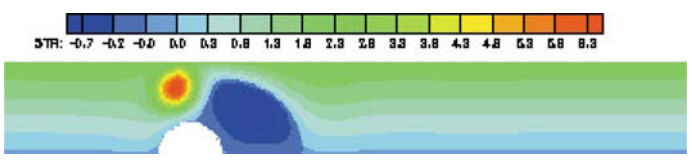

Fig. 6. Stream functions for Re = 10, B_o = 1.7 and Mn = 768.

The pressure contours at different magnetic field strengths are shown in Figs. 7, 8 and 9. It is clear that when the fluid is pulled toward the maximum magnetic field the pressure dropped. Once the fluid passed the thrombus it decelerates and the pressure increases and forms the recirculation zones.

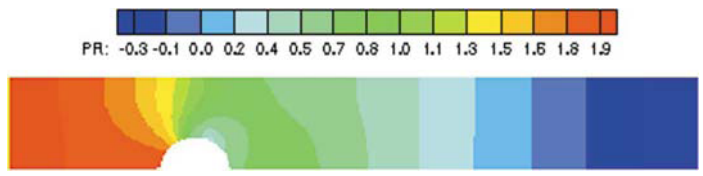

Fig. 7. Pressure contours for Re = 10 and B_o = 0.

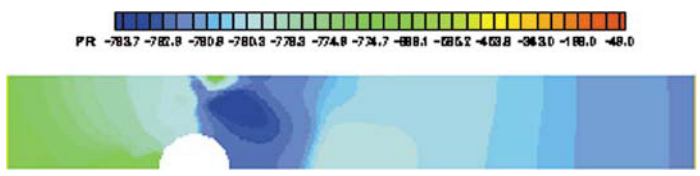

Fig. 8. Pressure contours for Re = 10, B_o = 1.3 and Mn = 449.

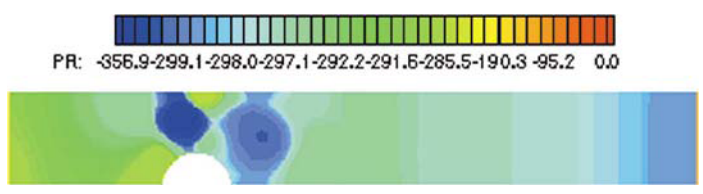

Fig. 9. Pressure contours for Re = 10, B_o = 1.7 and Mn = 768.

Figures 10 and 11 show the stream functions and Figures 12 and 13 show the pressure contours for Re = 100 and magnetic field of (B_o = 0, B_o = 1.3). Without magnetic field there is a small recirculation zone behind the thrombus. With the magnetic field the recirculation zone spreads and covers more area. However, no upstream separation or recirculation zone is formed. This is due to Reynolds number of 100 is ten times larger than Re = 10 which means 100 times more inertia force in this case. The magnetic field is attracting the fluid towards the base of the thrombus. When the inertia force is large it causes a formation of jet like stream near the thrombus. Another recirculation zone near the upper wall of the channel is formed.

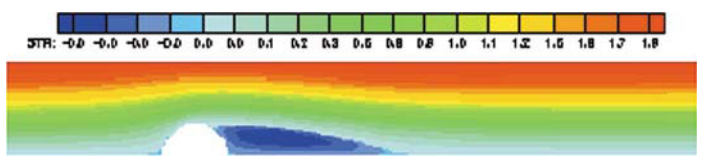

Fig. 10. Stream functions for Re = 100 and B_o = 0.

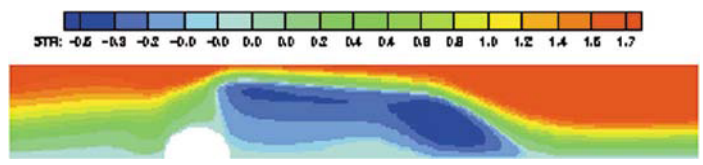

Fig. 11. Stream functions for Re = 100, B_o = 1.3 and Mn = 449.

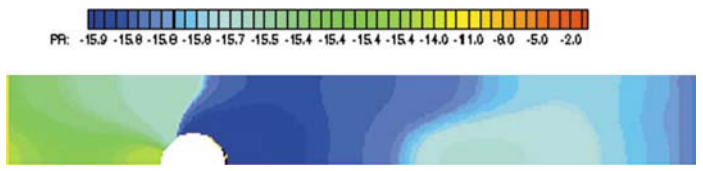

Fig. 12. Pressure contours for Re = 100 and B_o = 0.

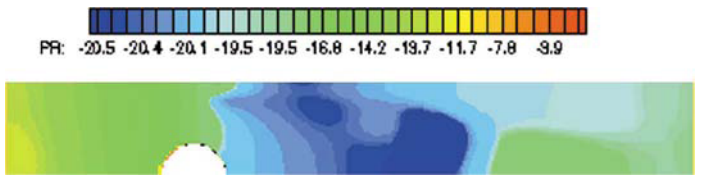

Fig. 13. Pressure contours for Re = 100, B_o = 1.3 and Mn = 449.

The friction coefficient is calculated based on

$$Cf = \frac{\tau_w}{\frac{1}{2}\rho V_o^2}$$

where τ_w is the wall shear stress. Figures 14 and 15 show the friction coefficient at the lower and upper walls of the channel for Re = 10 and different magnetic field strengths. It is evident that the friction coefficient increase drastically as the magnetic field increase from B_o = 0 to B_o = 1.3, and 1.7 tesla. The friction coefficient at the upper wall increase even higher than the lower wall.

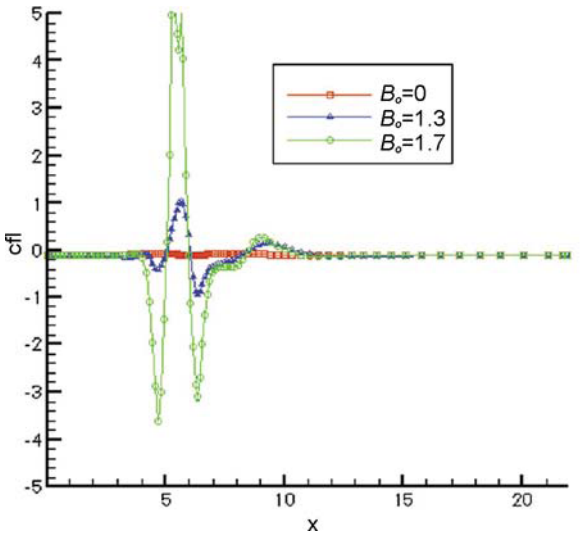

Fig. 14. Friction coefficient at lower wall at Re = 10, B_o = 0, 1.3 and 1.7.

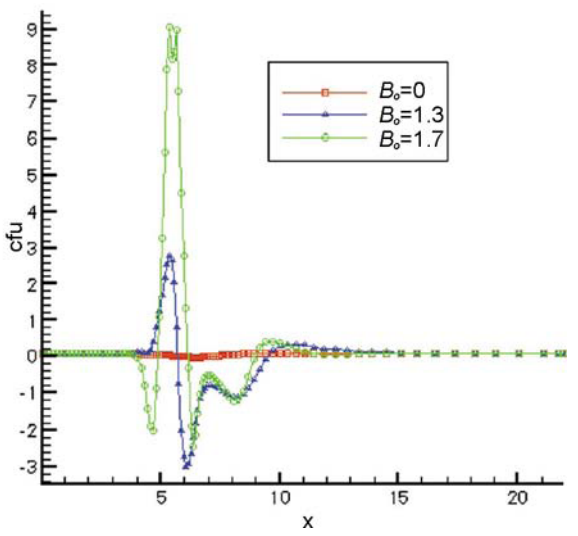

Fig. 15. Friction coefficient at upper wall at Re = 10, B_o = 0, 1.3 and 1.7.

The reason is the formation of the strong recirculation zones on top of the thrombus. For Re = 100 the friction coefficient is shown in Figs. 16 and 17 which show a plot of the friction coefficient at the lower and upper wall at Re = 100 for B_o = 0 and B_o = 1.3. When the magnetic field is on the friction coefficient at the lower wall is 10 times larger than without the magnetic field. However the friction coefficient remains in the same order of magnitude along the upper wall. This is because the fluid is rushed through the thrombus and forms a larger circulation zone behind the thrombus.

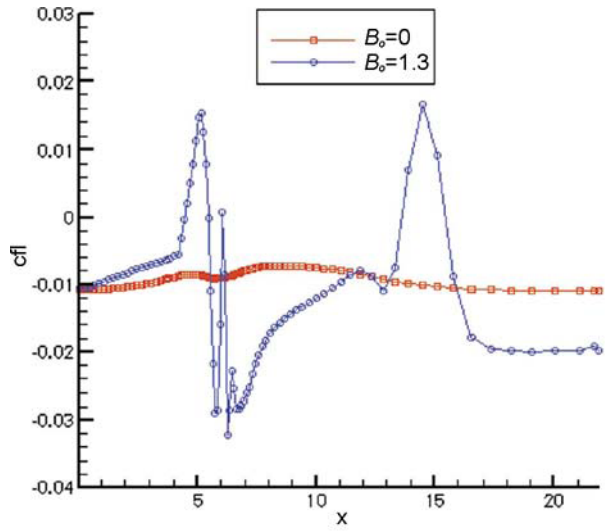

Fig. 16. Friction Coefficient at lower wall at Re = 100, B_o= 0, 1.3.

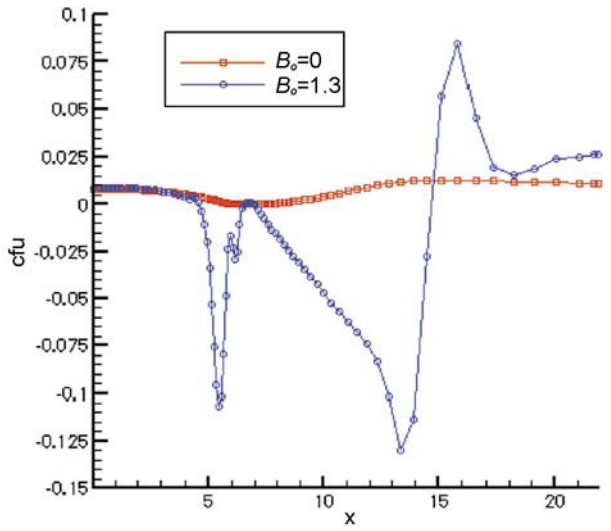

Fig. 17. Friction Coefficient at upper wall at Re = 100, B_o = 0, 1.3.

4. Conclusion

In the biomagnetic fluid dynamics the fluid is responding to the applied magnetic field through its magnetization property. Several medical applications that utilize the magnetic cell labeling where a magnetic material is attached to the cell membrane are being applied in clinical procedures. The magnetic particles are injected into the blood stream. This paper presents the biomagnetic fluid behavior in a two-dimensional channel with a semi-circular thrombus.

The finite analytic method with Cartesian grid is used to simulate the fluid behavior. It is found that when the magnetic field is on, the magnetic fluid flow behavior changes drastically. Formation of recirculation zones upstream of the thrombus causes the friction coefficient to increase.

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Author Profile

Yousef Haik: He received his B.Sc. (Eng) degree in Mechanical Engineering in 1986 from the University of Jordan, his M.Sc. degree in Mechanical Engineering in 1994 from the University of Iowa and his Ph.D. in Mechanical Engineering from the Florida State University in 1997. He is currently an associate scientist at the Florida A & M University-Florida State University College of Engineering. His research interests are in application of nanotechnology in microanalysis systems, biomagnetic fluid dynamics and computation fluid dynamics.

Ching-Jen Chen: He received his diploma in Mechanical Engineering in 1957 from Taipei Institute of Technology, his M.Sc. degree in Mechanical Engineering from Kansas State University in 1962 and his Ph.D. degree from Case Western Reserve University in Mechanical Engineering in 1967. He is currently the Dean of the Florida A & M University-Florida State University College of Engineering. His research interests are in computational fluid dynamics, microsystems, nanotechnology and biomagnetic fluid dynamics

Jhunu Chatterjee (nee Datta): She received her B.Sc. in 1987 in Chemistry from Presidency College under the University of Calcutta, India, Master of Technology in 1991 in Polymer Science and Engineering and Ph.D. in 1997 in Polymer Science and Engineering from the University of Calcutta, India. She worked in Kampala in East Africa for two years in a foam industry and in BDH Uganda Ltd. She joined as Post-doctoral fellow in the Department of Chemical Engineering at the Florida State University College of Engineering in 1998. At present she is a researcher in the Biomagnetic Engineering Lab at the FAMU-FSU College of Engineering. Her research interest is in synthesis and characterization of nanomagnetic materials for biomedical applications.